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EFFECT OF TURBULENCE ON THE CHARACTERISTICS OF A GLOW DISCHARGE

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A solution is obtained to the system of equations describing a glow discharge column in a turbulent gas stream. Calculations based on the given theory are found to agree closely enough with experimental data and with calculations made by other authors.

It is well known that turbulization of a flow improves the characteristics of a glow discharge: it allows the temperature of electrons as well as the electric field intensity and the discharge power to be raised, it ensures a higher stability of discharge, etc. [1-3]. It therefore seems worthwhile to develop methods of calculating the characteristics of glow discharge in a gas stream and to establish their dependence on the basic gasdynamic flow parameters. In this study there will be obtained an analytical solution to the problem pertaining to the positive column of such a discharge in a cylindrical channel.

As in the theory of a nonflow positive discharge column with diffusion [4], let the volume recombination be negligible and the temperature of electrons be uniform over the channel cross section with ionization occurring from the ground state of an atom upon collision with one electron. Furthermore, a complete correlation will be assumed [5] between the turbulence characteristics of the atomic gas and those of the ionic gas. The positive column of a glow discharge within the zone of fully turbulent flow can then be described by the equations

$$\frac{1}{Rr}\frac{d}{dr}\left(r\overline{n'u'}\right) = \frac{D_a}{R^2 r}\frac{d}{dr}\left(r\frac{d\bar{n}}{dr}\right) + \bar{n}\,\bar{z}_i, \tag{1}$$

$$\bar{n}U_i\bar{z}_i = \varkappa b_e\left(\bar{n}\,\bar{E}^2 + \bar{n}\,\overline{E'^2} + 2\bar{E}\,\overline{E'n'} + \overline{E'^2 n'}\right), \tag{2}$$

$$I = 2\pi R^2 e b_e \langle E\rangle \int_0^1 \bar{n} r\,dr \tag{3}$$

under the conditions

$$\bar{n}(1) = n'(1) = 0, \quad \frac{d\bar{n}}{dr}(0) = 0. \tag{4}$$

TABLE 1. Dependence of μ_1 and λ_1 on τ

τ	μ_1	γ_1
0	2,4048	0,2159
—10	1,4487	0,3015
—20	0,7413	0,3735
—30	0,3247	0,4182
—50	0,0461	0,4555

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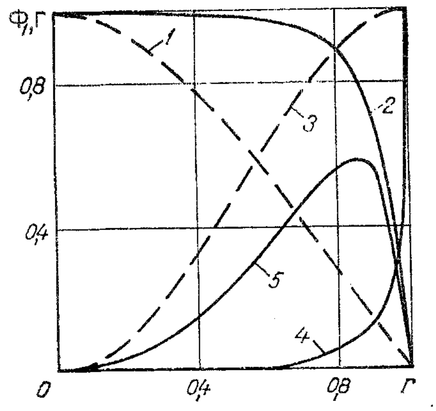

Fig. 1. Graphs of distributions $\Phi_1(\mu_1, r) = \bar{n}(r)/\bar{n}(0)$; $\Gamma_M = \frac{1}{\gamma_1(\mu_1 - \tau)\bar{n}(0)} \frac{d\bar{n}}{dr}$; $\Gamma_T = 2K\sqrt{\overline{u'^2}}\sqrt{\overline{n'^2}}\, U_i r / R\varkappa b_e \overline{E'^2}\, \bar{n}(0)$: 1, 3) $\tau = 0$; 2, 4, 5) $\tau = -30$.

Here <E> is some effective electric field intensity, which includes a constant component $\bar{E}$ and a fluctuating component E', and $\varkappa$ is the energy fraction lost by electrons during an ionizing collision. The last three terms on the right-hand side of Eq. (2), after being multiplied by the charge of an electron, represent components of Joule-effect dissipation due to turbulent fluctuations of the electric field intensity and of the charge density. According to relation (2), they should also represent a contribution to the formation of electron—ion pairs.

Introducing the correlation factor

$$\mathrm{K} = \overline{u'n'}/\sqrt{\overline{u'^2}}\sqrt{\overline{n'^2}} \tag{5}$$

and assuming that generation of electron—ion pairs as a result of turbulent Joule-effect dissipation is compensated by their turbulent diffusion, one can obtain from Eq. (1) the approximate distribution of rms charge density fluctuations which satisfies condition (4), viz.,

$$\sqrt{\overline{n'^2}} = \frac{\varkappa R b_e \overline{E'^2}}{2U_i\sqrt{\overline{u'^2}}\,\mathrm{K}}\, r\bar{n}. \tag{6}$$

The boundedness of $\overline{n'^2}$ at the channel axis dictates, moreover, that $2\bar{E}\overline{E'n'} + \overline{E'^2n'} = 0$. Expressions (5) and (6) reduce Eq. (1) to

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\bar{n}}{dr}\right) + \frac{\tau}{2} r\frac{d\bar{n}}{dr} + \mu^2\bar{n} = 0. \tag{7}$$

Here $\mu^2 = \varkappa b_e R^2 \bar{E}^2/U_i D_a$ and $\tau = -\varkappa R^2 b_e \overline{E'^2}/U_i D_a$ is the turbulence parameter expressed through the dispersion of the electric field intensity. The solution to Eq. (7) which will be bounded at $r = 0$ is

$$\bar{n}(r) = \bar{n}(0)\,\Phi_1(\mu_1, r),$$

$$\Phi_1(\mu_1, r) = 1 + \sum_{m=1}^{\infty} \frac{(-1)^m \mu_1^2(\mu_1 + \tau)(\mu_1^2 + 2\tau)\ldots[\mu_1^2 + (m-1)\tau]}{2^{2m}(m!)^2} r^{2m}, \tag{8}$$

TABLE 2. Data on T_e and z_i in a Positive Column of an Argon Plasma with $p = 3332$ N/m², $R = 1.1\cdot10^{-2}$ m, and $I = 1.8$ A

Conditions of the experiment			Numerical calculations [5]		This study		Experiment [2]	
N_{Re}	N_{Sc}	T, °K	T_e, eV	Z_i, 10^3 sec⁻¹	T_e, eV	Z_i, 10^3 sec⁻¹	T_e, eV	Z_i, 10^3 sec⁻¹
1000	4,0	603	0,93	3,62	1,05	6,36	1,04	5,0
1900	3,91	553	0,969	5,41	1,07	10,21	1,06	9,0
2400	3,87	533	0,981	6,24	1,09	12,12	1,07	10,2
3100	3,82	513	1,000	7,41	1,11	14,67	1,09	15,0
4000	3,77	488	1,017	8,70	1,13	17,63	1,11	19,0
4800	3,72	468	1,033	9,84	1,14	19,95	1,12	22,0
6300	3,62	443	1,057	11,88	1,18	24,51	1,13	28,3

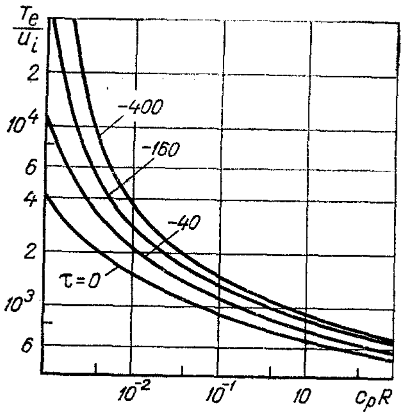

Fig. 2

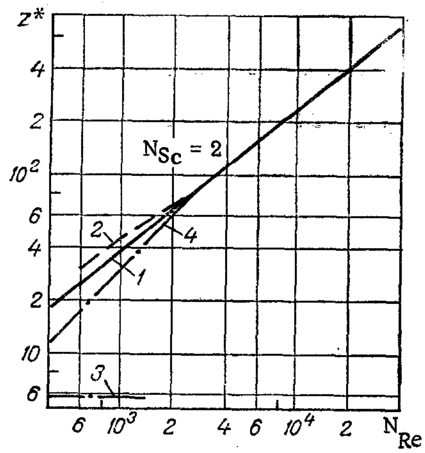

Fig. 3

Fig. 2. Dependence of T_e/U_i (°K/V) on cpR (torr·cm).

Fig. 3. Dependence of the relative ionization frequency $z^* = R^2\bar{z}_i/D_a$ on the Reynolds number N_{Re}: 1) calculation according to our theory; 2) numerical calculations [5]; 3) results based on the Schottky theory; 4) approximation [5] for the transition range of flow.

with μ_1 denoting the first root of the equation $\Phi(\mu) = 0$. Multiplying Eq. (7) by rdr and then integrating from 0 to 1 yields

$$-\frac{d\bar{n}}{dr}(1) = (\mu_1^2 - \tau)\gamma_1, \quad \gamma_1 = \int_0^1 \Phi_1(\mu_1, r)\, r dr. \tag{9}$$

Inasmuch as

$$-D_a \frac{d\bar{n}}{dr}(1) = R^2\bar{z}_i\gamma_1, \tag{10}$$

this together with expression (9) yields

$$\bar{z}_i = \frac{D_a}{R^2}(\mu_1^2 - \tau). \tag{11}$$

We now have for the electric field intensity $\langle E \rangle$, the charge density $\bar{n}(0)$ at the column axis, and the mean charge density $\bar{n}_c$

$$\langle E \rangle = \sqrt{\frac{U_i k T_e}{\varkappa R^2 e} \frac{b_i}{b_e} (\mu_1^2 - \tau)}, \tag{12}$$

$$\bar{n}(0) = \bar{I}/2\pi R^2 e b_e \langle E \rangle \gamma_1, \quad \bar{n}_c = \bar{I}/\pi R^2 e b_e \langle E \rangle \tag{13}$$

from relations (3) and (11). The expression for T_e [4] can, with the aid of relation (11), be rewritten as

$$1.16 \cdot 10^7 (cpR)^2 \sqrt{\frac{kT_e}{eU_i}} \left(1 + \frac{eU_i}{kT_e}\right) \exp\left(\frac{eU_i}{kT_e}\right) = \frac{\mu_1^2 - \tau}{\lambda_1^2}, \tag{14}$$

where λ_1 is the first root of the Bessel function of the zeroth order, p is the pressure, and constant c depends on the kind of gas. Integrating expression (6) yields

$$\tau = -2K\varepsilon^2 N_{Re} N_{Sc} N_{Pr,\,d}^{-1}, \tag{15}$$

where ε is the mean-over-the-radius degree of turbulence and N_{Re}, N_{Sc}, $N_{Pr,d}$ are, respectively, the Reynolds number, the Schmidt number, and the diffusional Prandtl number.

On the basis of the thus-derived expressions (6), (8), and (11)-(15), one can calculate the characteristics of discharge in a channel with a stabilized turbulent flow of plasma. The quantities μ_1, γ_1, and $\Phi_1(\mu_1, r)$ in these expressions have already been evaluated and tabulated earlier [6]. Here in Table 1 are given some values of μ_1 and γ_1, which indicate

TABLE 3. Data on E and $\bar{n}(0)$*

Numerical calculations [5]	This study		Experiment [2]	
E, 10^2 V/m	$\langle E\rangle$, 10^2 V/m	$\bar{n}(0)$, 10^{19} m^{-3}	E, 10^2 V/m	$\bar{n}(0)$, 10^{19} m^{-3}
0,8	1,58	3,31	1,05	4,26
0,97	2,14	2,77	1,25	4,26
1,03	2,4	2,61	1,35	3,84
1,11	2,73	2,45	1,49	3,67
1,22	3,12	2,33	1,59	3,53
1,30	3,42	2,26	1,68	3,36
1,48	3,98	2,14	1,74	3,16

*Conditions of the experiment the same as in Table 2.

the trend of their dependence on τ. At $\tau = 0$ function $\Phi_1(\mu_1, r)$ becomes $J_0(\lambda_1, r)$, $\mu_1 = \lambda_1$, and expressions (8), (11)-(13) yield those in the Schottky theory for a nonflow discharge column.

The graphs in Fig. 1 depict distributions of the relative charge density $\Phi_1(\mu_1, r)$ (curves 1 and 2), the ionization energy flux of molecular diffusion Γ_M (curves 3 and 4), and the ionization energy flux of turbulent diffusion Γ_T (curve 5). A comparison of curves 3 and 4 reveals that molecular diffusion in a turbulent positive column occurs only at the channel walls, while transfer of energy and charges within the central region is effected mainly by turbulent diffusion. Evidently, turbulent fluctuations cause the $\bar{n}(r)$ profile to become fuller and the gradient of $\bar{n}$ at the column boundary to increase in absolute value so that the ion current toward the channel wall increases. According to relations (12)-(14) and the data in Table 1, moreover, the electric field intensity increases while the charge density at the channel axis decreases approaching its mean value $\bar{n}_c$ as $|\tau|$ increases. The increasing loss of electron—ion pairs from the discharge column is compensated by an increasing ionization frequency $\bar{z}_i$, as follows from relation (11). An increase of frequency $\bar{z}_i$ is effected by raising the electron temperature of the plasma.

The dependence of T_e/U_i on cpR is shown in Fig. 2 for several values of τ, based on expression (14). It appears that at small values of cpR the electron temperature T_e can increase appreciably with an increasing degree of turbulence. Thus flow turbulization causes T_e, $\bar{z}_i$, and $\langle E\rangle$ to increase, the distribution of current density over the channel cross section to become more uniform and, consequently, transition from diffusive discharge to confined discharge to occur at higher threshold current. These conclusions, based on the analytical solution, have been confirmed by well-known experimental and theoretical data [1-5]. We will now compare the results of calculations according to the relations derived in this study with the experimental data in [2] and with the results of numerical calculations in [5].

In a positive column the fluctuations of velocity and charge density occur simultaneously so that one can assume $K = 1$. The value of the Prandtl number $N_{Pr,d}$ lies within the 0.5-0.75 range [7]. Taking the middle value $N_{Pr,d} = 0.63$ and using the equalities [8] $\varepsilon^2 = \zeta/8$, $\zeta = 0.184 N_{Re}^{-0.2}$ (for a channel with smooth walls), we obtain

$$\tau = -0.073\, N_{Re}^{0.8} N_{Sc}. \qquad (16)$$

The mobility of particles was calculated according to the relation $b_\alpha = ekT_\alpha/\sqrt{2} m_\alpha Q_{\alpha a} \bar{v}_\alpha p$, with $\bar{v}_\alpha = \sqrt{8kT_\alpha/\pi m_\alpha}$, and $\alpha = i$ or e. From the data on cross sections for collisions in argon [4, 9], $Q_{ea} = 0.66 \cdot 10^{-23}\, T_e$ and $Q_{ia} = 7.5 \cdot 10^{-19}$ (m^2) could be assumed for the $T_e = (5-30) \cdot 10^3$ °K range.

The authors of another study [5] have generalized their numerical data on the ionization frequency in a turbulent positive column by the expression

$$\frac{R^2}{D_a} z_i = \lambda_1^2 + 0.08\, N_{Re}^{0.8} N_{Sc}^{0.875}. \qquad (17)$$

According to the curves in Fig. 3, depicting the dependence of the relative ionization frequency on the Reynolds number, calculations based on our theory agree almost exactly with the calculations by those authors [5].

Values of T_e and $\bar{z}_i$ based on relation (11) and obtained, with the aid of the graph in Fig. 2, for various values of N_{Re}, N_{Sc}, and temperatures T of heavy particles are given in Table 2 alongside data from the other two studies [2, 5]. The results of both numerical and analytical calculations are obviously in a satisfactory agreement with experimental results

In order to calculate <E>, and $\bar{n}$ it is necessary to know $\varkappa$. An evaluation of the data in [2] on the basis of the relation $\varkappa = 2\pi R^2 eU_i z_i \bar{n}(0)\gamma_1/IE$ revealed that the minimum value of $\varkappa$ under conditions of the experiment [2] had been approximately 0.5 and then, with higher values of N_{Re}, would increase to approximately 1. The intermediate value, $\varkappa = 0.7$ (for the range of N_{Re} values in Table 2), was used in our calculations.

An analysis of <E> and $\bar{n}(0)$ values in Table 3 indicates a satisfactory agreement between analytical calculations of $\bar{n}(0)$ and experimental data, while values of <E> obtained by expression (12) exceed the values obtained in [2] and in [5] on the average by 90 and 140%, respectively. The lower values of E in [5] can be explained by the method of calculating the electric field intensity there without taking into account the turbulent diffusion of charges. The higher values of <E> according to relation (12) than those obtained experimentally can be explained by a difference between the gas composition assumed for calculations (pure argon) and the actual gas composition in the experiment [2].

We have thus obtained a solution to the system of equations describing a glow discharge column in a turbulent gas stream. The results of the Schottky theory follow from it, as a special case. The characteristics of a positive column calculated according to the expressions given here agree with the experimental data in [2] and with the results of numerical calculations in [5].

In [5] the electric field intensity was calculated according to the expression

$$E = \sqrt{\frac{U_i k T_e b_i}{\varkappa R^2 b_e e}\lambda_1^2},$$

inconsistent with calculations of z_i. Indeed, since $U_i z_i = \varkappa b_e E^2$, and expression (17) for z_i has been obtained in [5], it must be that

$$E = \sqrt{\frac{U_i k T_e b_i}{\varkappa R^2 b_e e}\left(\lambda_1^2 + 0.08\, N_{Re}^{0.8} N_{Sc}^{0.875}\right)},$$

and this is almost identical to expression (12).

One can expect that more intricate models of glow discharge will also yield satisfactory results, if the turbulent diffusion of electron–ion pairs is assumed to be equal to their generation as a result of turbulent Joule-effect dissipation.

NOTATION

I, current; E, electric field intensity; u, radial component of velocity; D_a, coefficient of ambipolar diffusion; U_i, ionization potential; b, mobility of particles; n, charge density; z_i, ionization frequency; e, charge of an electron; T, temperature; k, Stefan–Boltzmann constant; $Q_{\alpha a}$, cross section for particle–atom collision; v_α, velocity of particles; ζ, hydraulic drag coefficient; J_0, Bessel function of the zeroth order; R, radius of a positive column; r, radial coordinate referred to R; superscripts: a prime, fluctuation value, and a dash denotes the mean value; subscripts: i, ions and e, electrons.

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DIFFUSION OF HYDROGEN IN HAFNIUM AND TITANIUM

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We measured the coefficients of diffusion of hydrogen in the hydride phases of hafnium and titanium at 1073-1273°K on the basis of the solutions of Fick's second law for diffusion in a finite cylinder and in a sector of it.

Hafnium is one of those scattered rare metals which have become important in industry during the past 25 years. In the interaction of hydrogen with hafnium, at least three phases are formed: if the hydrogen content is low, there is an α-solid solution; in addition, there are two other phases of variable composition — a nonstoichiometric dihydride and a β-solid solution. Data on hydrogen diffusion are necessary for the establishment of well-grounded conditions in the thermovacuum processing of metals, the production of hafnium hydrides, the manufacture of parts made from these, etc. [1, 2].

In the present study, we investigated the diffusion of hydrogen by the method of desorption into a vacuum with mass-spectrometric recording. The metal specimens consisted of disks of "iodide" hafnium 1 cm in diameter and 0.4 cm thick; after cutting, they were washed with ethyl alcohol and air-dried. The experiments were conducted on a vacuum main (Fig. 1) combined with the admittance system of an MI-1305 mass spectrometer.

To calculate the diffusion coefficients, we used Fick's second law, and our solution for the diffusion of a gas in a finite cylinder for zero boundary conditions at constant initial concentration C_0 has the form

$$C(r,\ \varphi,\ z,\ t)=\frac{16C_0}{\pi}\sum_{\substack{i=1\\m=0}}^{\infty}\frac{J_0\left(\frac{\mu_0^i}{R}r\right)}{(2m+1)(\mu_0^i)^2}\sin\frac{\pi(2m+1)}{l}z\exp\left\{-\left[\left(\frac{\mu_0^i}{R}\right)^2+\frac{\pi^2(2m+1)^2}{l^2}\right]Dt\right\}. \quad (1)$$

The zero boundary conditions were ensured by constant pumping of the gas from the volume V above the specimen. Taking account of the fact that the experimentally measured quantity was not the concentration of the gas inside the specimen but the pressure of the gas generated, we solved the differential equation for the "balance" of the gas [3]

$$dQ_{rel}/dt=V\cdot dp/dt+pF \quad (2)$$

and obtained

$$p=\frac{1}{V}\exp\left(-\frac{F}{V}t\right)\int\frac{dQ_{rel}}{dt}\exp\left(\frac{F}{V}t\right)dt. \quad (3)$$

Since $Q_{rel}=Q_0-Q(r,\ \varphi,\ z,\ t)$, it follows that $dQ_{rel}/dt=-dQ(r,\ \varphi,\ z,\ t)/dt$. The quantity $Q(r,\ \varphi,\ z,\ t)$, in turn, is given by the formula

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